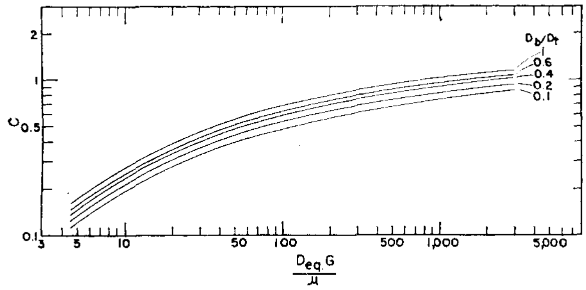

Fig. 3. Design chart for ball flow meters.

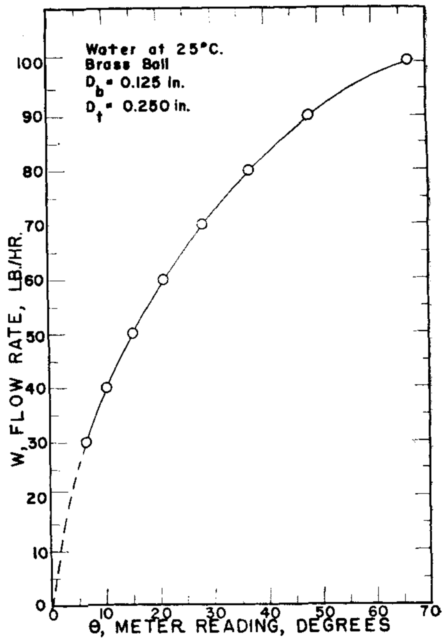

Fig. 4. Calibration curve for sample design.

and the corresponding values of Deq. G/μ are calculated. From Figure 3 the values of C, corresponding to a D_b/D_t of 0.5 and the appropriate values of Deq. G/μ, are obtained. Equation (4) or (5) may now be used to obtain values of sin θ, from which θ is obtained. The calibration curve is shown on Figure 4.

NOTATION

A_b = projected cross-sectional area of ball, sq. ft.
A_f = free area for flow of fluid between ball and tube, sq. ft.
C = ball-flow-meter coefficient, dimensionless
C_d = drag coefficient for spheres, dimensionless
D_b = ball diameter, ft.
D_b' = ball diameter, in.
Deq. = equivalent diameter for use in the Reynolds number, $(D_t - D_b)$, ft.
D_t = tube diameter, ft.
D_t' = tube diameter, in.
G = mass flow rate based on free area (A_f), lb./(sec.)(sq. ft.)
g_c = conversion factor, (lb.-mass)(ft.)/(lb. force)(sec.)(sec.)
T = temperature of fluid, °C.
u = fluid velocity, ft./sec.
v_b = volume of ball, cu. ft.
W = fluid flow rate, lb./sec.

Greek Letters

θ = meter reading, angle from vertical, deg.
μ = fluid viscosity, lb./(ft.)(sec.)
ρ_b = ball density, lb./cu. ft.
ρ_f = fluid density, lb./cu. ft.
τ = drag on sphere per unit of projected area, lb.-force/sq. ft.

LITERATURE CITED

1. Goldstein, S., ed., "Modern Developments in Fluid Dynamics," Oxford Univ. Press, London (1938).
2. Gradishar, F. J., *Chem. Eng.*, 57, No. 10, 122 (1950).
3. Shulman, H. L., F. M. Stieger, and A. R. Leist, *ibid.*, 60, No. 2, 173 (1953).

Absorption into an Accelerating Film

L. E. SCRIVEN and R. L. PIGFORD

University of Delaware, Newark, Delaware

If the velocity field in the neighborhood of the gas-liquid interface is not uniform in a steady state apparatus such as a wetted-wall column or a jet, then the rate of absorption is no longer given by the simplest form of the so-called "penetration" theory, in which the local absorption rate is assumed to be the stagnant-liquid absorption rate. Rather, a more general form of the diffusion equation must be employed in which it may not be especially convenient to replace contact distance by contact time. Since this matter does not seem to have been fully allowed for by the author of a recent communication (*3*), a brief discussion is given here. For the purpose of illustration, the magnitude of the acceleration end effect in a short wetted-wall column is estimated from the equations presented herein. Such an end effect was suggested by Vivian and Peaceman (*4*) as a possible explanation of the discrepancy between experimental and theoretical absorption rates in their columns.

The equation governing ordinary diffusion in a binary system, for constant diffusivity and mass density, is

$$\frac{\partial c}{\partial t} + (\mathbf{u} \cdot \nabla)c = D\nabla^2 c \qquad (1)$$

For a short wetted-wall column or jet it is generally permissible to regard the absorbing liquid as two dimensional with infinite depth and to neglect diffusion in the direction of flow; at steady state Equation (1) then reduces to

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \qquad (2)$$

where c is concentration in mass per unit volume, x and u are distance and the velocity component parallel to the interface respectively, y and v are distance and the velocity component normal to the interface respectively, and D is the diffusivity. The appropriate boundary conditions are

$$c = c_0 \quad \text{for} \quad x < 0, \quad y \geq 0$$

$$c = c_0 \quad \text{for} \quad x > 0, \quad y = \infty$$

$$c = c_e \quad \text{for} \quad x \geq 0, \quad y = 0$$

where interfacial equilibrium and absence both of heat effects at the interface and of gas-phase diffusional resistance have been assumed. At the interface $(\partial u/\partial y)_{x,0}$ will be very nearly zero (the adjoining gas exerts negligible drag upon the liquid surface), and so u may be replaced by the surface velocity u_s, provided that the penetration depth of solute gas molecules is sufficiently small. From the equation of continuity

$$v = -\int_0^y \frac{\partial u}{\partial x}\,dy \qquad (3)$$

Hence in the region of interest, close to the interface, the normal component of velocity is approximated by

$$v = -y\frac{du_s}{dx} \qquad (4)$$

and the diffusion equation becomes

$$u_s\frac{\partial c}{\partial x} - y\frac{du_s}{dx}\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \qquad (5)$$

If the liquid is being accelerated in the x direction, liquid elements are stretched in the x direction and suffer a corresponding shrinkage in the y direction; this causes a bulk flow of liquid toward the surface, which in the case of absorption opposes the mass flux due to the concentration gradient, but at the same time results in a steepening of the concentration gradient. The second term in Equations (2) and (5) represents the net effect of bulk flow normal to the surface.

In the special case of constant surface velocity Equation (5) reduces to

$$u_s \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial y^2} \qquad (6)$$

With the substitution of $\theta = x/u_s$ this becomes the familiar equation for diffusion in a stagnant liquid, corresponding to which the local absorption rate per unit area is

$$N_A^* \quad \Delta c\sqrt{\frac{Du_s}{\pi x}} = \Delta c\sqrt{\frac{D}{\pi\theta}} \qquad (7)$$

Toor (3) considered the case in which

$$u_s = ax^m, \quad -1 < m < 1 \qquad (8)$$

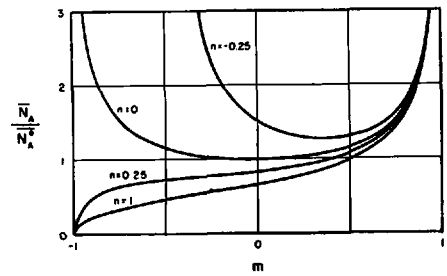

Fig. 1. Average absorption rates for varying surface velocity [Equation (15)].

and attempted to evaluate the effect of variable perimeter and surface velocity on average absorption rate from

$$\overline{N_A} = \frac{\int_0^X N_A(x)P(x)\,dx}{\int_0^X P(x)\,dx} = \frac{\int_0^\theta N_A P u_s\,d\theta}{\int_0^\theta P u_s\,d\theta} \qquad (9)$$

His result appears to be incomplete, inasmuch as he made use in Equation (9) of N_A^* calculated from Equation (6) instead of the true value, N_A, from Equation (5). The correct result is obtained by substituting Equation (8) in (5) and solving by a method given in reference *2* to obtain

$$N_A = \Delta c\sqrt{\frac{a(1+m)D}{\pi x^{1-m}}} \qquad (10)$$

$$= \sqrt{(1+m)/(1-m)}\,N_A^*(\theta) \qquad (11)$$

In an evaluation of N_A^* according to Equation (7), the time θ is taken as

$$\theta = \int_0^x u_s^{-1}\,dx \qquad (12)$$

If the perimeter $P(x)$ varies according to

$$P = bx^n, \quad n > -(1+m)/2 \qquad (13)$$

the average absorption rate, if use is made of Equation (9), is

$$\overline{N_A} = \Delta c \frac{2(1 + n)}{(1 + m + 2n)} \cdot \sqrt{\frac{a(1 + m)D}{\pi(X)^{1-m}}} \quad (14)$$

$$= \frac{1 + n}{1 + m + 2n} \cdot \sqrt{\frac{1 + m}{1 - m}} \overline{N_A^*(\theta')} \quad (15)$$

where $\overline{N_A^*(\theta')}(= 2\Delta c \sqrt{D/\pi\theta'})$ is the average absorption rate into a film of constant perimeter moving at constant velocity over the same total distance. The ratio $\overline{N_A}/\overline{N_A^*}$ is shown in Figure 1.

In flow down a wetted-wall column like the columns employed by Vivian and Peaceman (*4*) the film surface must accelerate from zero velocity at the inlet slot and approach the final steady velocity downstream. In the past it has been assumed that the final velocity is attained immediately at the inlet slot. For the purpose of estimating the effect on average absorption rate of the acceleration end effect, it will be assumed that the outermost portion of the film accelerates downward under the influence of gravity alone until it reaches the final steady velocity, with which it continues to move thereafter. During the acceleration period ($0 < x \leq x_a$) the surface velocity is approximately the free-fall velocity,

$$u_s = \sqrt{2g_L}x^{1/2} \quad (16)$$

and the local absorption rate, from Equation (10), is

$$N_{Aa} = \Delta c\sqrt{\frac{3D}{\pi}} \left(\frac{g_L}{2x}\right)^{1/4}, \quad 0 < x \leq x_a \quad (17)$$

At the end of the acceleration period the concentration within the film is given by (*1*)

$$c = c_0 + \Delta c \operatorname{erfc}\left[\frac{y}{2}\sqrt{\frac{3}{D}}\left(\frac{g_L}{2x_a}\right)^{1/4}\right] \quad (18)$$

$$x_a = u_f^2/2g_L \quad (19)$$

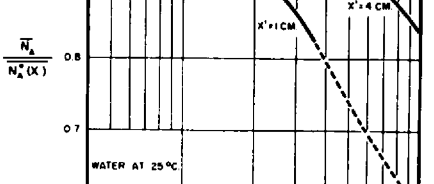

Fig. 2. Influence of acceleration end effect on average rate of absorption in short wetted-wall columns [Equation (24)].

where u_f is the final steady velocity (*4*). At the end of a fictitious distance x_a' the concentration within the film, if it moved with constant velocity u_f, would have been

$$c = c_0 + \Delta c \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{u_f}{Dx_a'}}\right) \tag{20}$$

Thus, as far as subsequent absorption is concerned, the free-fall distance x_a can be replaced by an equivalent distance x_a' given by

$$x_a' = u_f^2/3g_L \tag{21}$$

The average absorption rate over the entire contact distance X is found by means of Equation (9):

$$\overline{N_A} = (X)^{-1}\int_0^{x_a} N_{Aa}\,dx + (X)^{-1}\int_{x_a'}^{x_a'+X-x_a} N_A^*(x)\,dx \tag{22}$$

$$= 2\Delta c\sqrt{\frac{Du_f}{\pi X}}\left(1 - \frac{x_a}{3X}\right), \quad x_a \leq X \tag{23}$$

Comparison of this result with the average absorption rate if it is assumed that the final velocity is immediately attained leads to (*1*)

$$\frac{\overline{N_A}}{N_A^*(X)} = \sqrt{1 - \left(\frac{4\Gamma}{\mu}\right)\frac{\delta}{32X}}, \quad \left(\frac{4\Gamma}{\mu}\right)\frac{\delta}{32X} \leq \frac{1}{3} \tag{24}$$

where Γ is the mass flow rate of liquid per unit perimeter, δ the film thickness, and μ the liquid viscosity.

This analysis indicates that the error in assuming that (7) applies to short wetted-wall columns 1 to 6 cm. long may lead to much too high predictions of absorption rates when the Reynolds number $4\Gamma/\mu$ exceeds 200 to 500 (see Figure 2; operation has generally been in the range 100 to 1,200). The simplifying assumptions which have been made here are of course open to question. For instance, the shear force exerted by the supporting wall will cause the surface to accelerate a little more slowly than in free fall, but on the other hand the flow and geometry of the film near the inlet may be such that the initial downward velocity of the surface is greater than zero.

LITERATURE CITED

1. Scriven, L. E., Ph.D. thesis, Univ. Delaware, Newark (1956).
2. ———, and R. L. Pigford, to be published.
3. Toor, H. L., *A.I.Ch.E. Journal*, **2**, 578 (1956).
4. Vivian, J. E., and D. W. Peaceman, *ibid.*, 437.